A Coin Partitioning Problem

Sabih H. Gerez
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Problem: In how many ways can one distribute n coins among x people, where everybody gets at least one coin, where persons can be distinguished but coins cannot.

Solution

I have used dynamic programming. Let's call the number of ways for n coins and x people $W(n,x)$. Then:

$$W(n,x) = W(n-1,x) + W(n-1,x-1)$$

This can be understood as follows. Suppose that we know the solution for $n-1$ coins. For n coins, the first person can either have 2 coins or more or exactly 1 coin. The number of solutions where n coins are distributed across x people with the restriction that the first person has at least two coins, is the same as the situation where $n-1$ coins are distributed across x people with the first person having at least one coin. This is expressed in the first term. The number of solutions where n coins are distributed across x people with the restriction that the first person has only 1 coin, is the same as in the situation where $n-1$ coins are distributed across $x-1$ people. This is expressed in the second term. With the boundary condition that $W(n,n) = 1$ for all n, all values of $W(n,x)$ can be computed recursively.

When the numbers are calculated and put in a two-dimensional arrangement, one sees Pascal's triangle. This directly leads to a closed-form solution:

$$W(n,x) = \binom{n-1}{x-1} = \frac{(n-1)!}{(x-1)!(n-x)!}$$

It is now obvious that $W(11,3) = 45$.